**Section 8-6
Testing a Claim About a
Standard Deviation or
Variance**

Requirements for Testing Claims About σ or σ^2

n = sample size

s = *sample* standard deviation

s^2 = *sample* variance

σ = claimed value of the *population* standard deviation

σ^2 = claimed value of the *population* variance

8.1 - 2

1

Requirements for Testing Claims About σ or σ^2

1. The sample is a simple random sample.

2. The population has a normal distribution. (This is a much stricter requirement than the requirement of a normal distribution when testing claims about means.)

8.1 - 3

Chi-Square Distribution

Test Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

8.1 - 4

P-Values and Critical Values for Chi-Square Distribution

- Use Table A-4.
- The degrees of freedom = *n* –1.

2

8.1 - 5

Properties of Chi-Square Distribution

- All values of χ^2 are nonnegative, and the distribution is not symmetric (see Figure 8-13, following).

- There is a different distribution for each number of degrees of freedom (see Figure 8-14, following).

- The critical values are found in Table A-4 using *n* – 1 degrees of freedom.

8.1 - 6

Properties of Chi-Square Distribution - cont

Properties of the Chi-Square Distribution
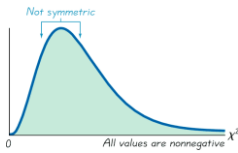
Figure 8-13

Chi-Square Distribution for 10 and 20 df
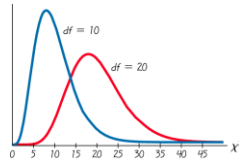
Different distribution for each number of df.

Figure 8-14

8.1 - 7

Table A-4

Table A-4 is based on cumulative areas from the right (unlike the entries in Table A-2, which are cumulative areas from the left). Critical values are found in Table A-4 by first locating the row corresponding to the appropriate number of degrees of freedom (where df = n –1). Next, the significance level α is used to determine the correct column. The following examples are based on a significance level of α = 0.05, but any other significance level can be used in a similar manner.

8.1 - 8

3

Table A-4

Right-tailed test:

Because the area to the right of the critical value is 0.05, locate 0.05 at the top of Table A-4.

Left-tailed test:

With a left-tailed area of 0.05, the area to the right of the critical value is 0.95, so locate 0.95 at the top of Table A-4.

8.1 - 9

Table A-4

Two-tailed test:

Unlike the normal and Student t distributions, the critical values in this χ^2 test will be two different positive values (instead of something like ±1.96). Divide a significance level of 0.05 between the left and right tails, so the areas to the right of the two critical values are 0.975 and 0.025, respectively. Locate 0.975 and 0.025 at the top of Table A-4

8.1 - 10

Example:

A common goal in business and industry is to improve the quality of goods or services by reducing variation. Quality control engineers want to ensure that a product has an acceptable mean, but they also want to produce items of consistent quality so that there will be few defects. If weights of coins have a specified mean but too much variation, some will have weights that are too low or too high, so that vending machines will not work correctly (unlike the stellar performance that they now provide).

8.1 - 11

4

Example:

Consider the simple random sample of the 37 weights of post-1983 pennies listed in Data Set 20 in Appendix B. Those 37 weights have a mean of 2.49910 g and a standard deviation of 0.01648 g. U.S. Mint specifications require that pennies be manufactured so that the mean weight is 2.500 g. A hypothesis test will verify that the sample appears to come from a population with a mean of 2.500 g as required, but use a 0.05 significance level to test the claim that the population of weights has a standard deviation less than the specification of 0.0230 g.
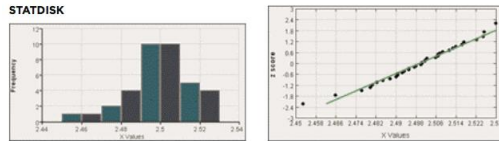
8.1 - 12

Example:

Requirements are satisfied: simple random sample; and STATDISK generated the histogram and quantile plot - sample appears to come from a population having a normal distribution.

STATDISK

Example:

Step 1: Express claim as $\sigma < 0.0230$ g

Step 2: If $\sigma < 0.0230$ g is false, then $\sigma \geq 0.0230$ g

Step 3: $\sigma < 0.0230$ g does not contain equality so it is the alternative hypothesis; null hypothesis is $\sigma = 0.0230$ g

$$H_0: \sigma = 0.0230 \text{ g}$$
$$H_1: \sigma < 0.0230 \text{ g}$$

Step 4: significance level is $\alpha = 0.05$

Step 5: Claim is about σ so use chi-square

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Example:

Step 6: The test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(37-1)(0.01648)^2}{0.0230^2} = 18.483$$

The critical value from Table A-4 corresponds to 36 degrees of freedom and an "area to the right" of 0.95 (based on the significance level of 0.05 for a left-tailed test). Table A-4 does not include 36 degrees of freedom, but Table A-4 shows that the critical value is between 18.493 and 26.509. (Using technology, the critical value is 23.269.)
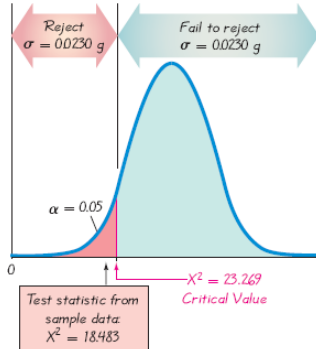
Example:

Example:

Step 7: Because the test statistic is in the critical region, reject the null hypothesis.

There is sufficient evidence to support the claim that the standard deviation of weights is less than 0.0230 g. It appears that the variation is less than 0.0230 g as specified, so the manufacturing process is acceptable.

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Recap

In this section we have discussed:

❖ **Tests for claims about standard deviation and variance.**

❖ **Test statistic.**

❖ **Chi-square distribution.**

❖ **Critical values.**

Chapter 9
Inferences from Two Samples

Section 9-1
Review and Preview

7

Review

In Chapters 7 and 8 we introduced methods of *inferential statistics*. In Chapter 7 we presented methods of constructing confidence interval estimates of population parameters. In Chapter 8 we presented methods of testing claims made about population parameters. Chapters 7 and 8 both involved methods for dealing with a sample from a single population.

Preview

The objective of this chapter is to extend the methods for estimating values of population parameters and the methods for **testing hypotheses** to situations involving two sets of sample data instead of just one.

The following are examples typical of those found in this chapter, which presents methods for using sample data from two populations so that inferences can be made about those populations.

Preview

- Test the claim that when college students are weighed at the beginning and end of their freshman year, the differences show a mean weight gain of 15 pounds (as in the "Freshman 15" belief).
- Test the claim that the proportion of children who contract polio is less for children given the Salk vaccine than for children given a placebo.
- Test the claim that subjects treated with Lipitor have a mean cholesterol level that is lower than the mean cholesterol level for subjects given a placebo.

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Section 9-2
Inferences About Two Proportions

Key Concept

In this section we present methods for (1) testing a claim made about the two population proportions and (2) constructing a confidence interval estimate of the difference between the two population proportions. This section is based on proportions, but we can use the same methods for dealing with probabilities or the decimal equivalents of percentages.

8.1 - 25

Notation for Two Proportions

For population 1, we let:

p_1 = **population** proportion

n_1 = size of the sample

x_1 = number of successes in the sample

$\hat{p}_1 = \dfrac{x_1}{n_1}$ (the **sample** proportion)

$\hat{q}_1 = 1 - \hat{p}_1$

The corresponding notations apply to

$p_2, n_2, x_2, \hat{p}_2,$ and $\hat{q}_2$, which come from population 2.

8.1 - 26

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Pooled Sample Proportion

❖ **The pooled sample proportion**
is denoted by $\bar{p}$ and is given by:

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

❖ We denote the complement of $\bar{p}$ by $\bar{q}$,
so $\bar{q} = 1 - \bar{p}$

8.1 - 27

Requirements

1. We have proportions from two **independent** simple random samples.

2. For each of the two samples, the number of successes is at least 5 and the number of failures is at least 5.

Test Statistic for Two Proportions

For $H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$, $H_1: p_1 < p_2$, $H_1: p_1 > p_2$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}}$$

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Test Statistic for Two Proportions - cont

For $H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$, $H_1: p_1 < p_2$, $H_1: p_1 > p_2$

where $p_1 - p_2 = 0$ (assumed in the null hypothesis)

$$\hat{p}_1 = \frac{x_1}{n_1} \quad \text{and} \quad \hat{p}_2 = \frac{x_2}{n_2}$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} \qquad \text{and} \qquad \bar{q} = 1 - \bar{p}$$

Test Statistic for Two Proportions - cont

P-value: Use Table A-2. (Use the computed value of the test statistic z and find its *P*-value by following the procedure summarized by Figure 8-5 in the text.)

Critical values: Use Table A-2. (Based on the significance level α, find critical values by using the procedures introduced in Section 8-2 in the text.)

Confidence Interval Estimate of $p_1 - p_2$

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$$

$$\text{where} \quad E = z_{\alpha/2}\sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

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Example:

The table below lists results from a simple random sample of front-seat occupants involved in car crashes. Use a 0.05 significance level to test the claim that the fatality rate of occupants is lower for those in cars equipped with airbags.

	Airbag Available	No Airbag Available
Occupant Fatalities	41	52
Total number of occupants	11,541	9,853

Example:

Requirements are satisfied: two simple random samples, two samples are independent; Each has at least 5 successes and 5 failures (11,500, 41; 9801, 52). Use the *P*-value method.

Step 1: Express the claim as $p_1 < p_2$.

Step 2: If $p_1 < p_2$ is false, then $p_1 \geq p_2$.

Step 3: $p_1 < p_2$ does not contain equality so it is the alternative hypothesis. The null hypothesis is the statement of equality.

Example:

H_0: $p_1 = p_2$ H_a: $p_1 < p_2$ (original claim)

Step 4: Significance level is 0.05

Step 5: Use normal distribution as an approximation to the binomial distribution. Estimate the common values of p_1 and p_2 as follows:

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{41 + 52}{11{,}541 + 9{,}853} = 0.004347$$

With $\bar{p} = 0.004347$ **it follows** $\bar{q} = 0.995653$

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Example:

Step 6: Find the value of the test statistic.

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\overline{pq}}{n_1} + \dfrac{\overline{pq}}{n_2}}}$$

$$= \frac{\left(\dfrac{41}{11{,}541} - \dfrac{52}{9{,}853}\right) - 0}{\sqrt{\dfrac{(0.004347)(0.995653)}{11{,}541} + \dfrac{(0.004347)(0.995653)}{9{,}853}}}$$
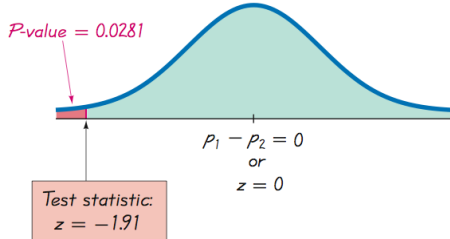
$$z = -1.91$$

Example:

P-value = 0.0281

$p_1 - p_2 = 0$
or
$z = 0$

Test statistic:
$z = -1.91$

Left-tailed test. Area to left of $z = -1.91$ is 0.0281 (Table A-2), so the P-value is 0.0281.

Example:

Step 7: Because the P-value of 0.0281 is less than the significance level of $\alpha = 0.05$, we reject the null hypothesis of $p_1 = p_2$.

Because we reject the null hypothesis, we conclude that there is sufficient evidence to support the claim that the proportion of accident fatalities for occupants in cars with airbags is less than the proportion of fatalities for occupants in cars without airbags. Based on these results, it appears that airbags are effective in saving lives.

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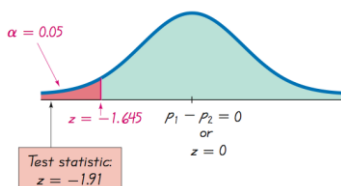## Example: Using the Traditional Method

With a significance level of $\alpha = 0.05$ in a left-tailed test based on the normal

$\alpha = 0.05$

$z = -1.645$ $p_1 - p_2 = 0$
or
$z = 0$

Test statistic:
$z = -1.91$

distribution, we refer to Table A-2 and find that an area of $\alpha = 0.05$ in the left tail corresponds to the critical value of $z = -1.645$. The test statistic of does fall in the critical region bounded by the critical value of $z = -1.645$.
We again reject the null hypothesis.